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3 (Sem-6/CBCS) STA HC 2

2022

STATISTICS

(Honours)

Paper : STA-HC-6026

**(Multivariate Analysis and
Nonparametric Analysis)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer **any seven** of the following questions
as directed : $1 \times 7 = 7$

(a) The moment generating function of bivariate normal distribution with parameters $(0, 0, \sigma_1^2, \sigma_2^2, \rho)$ is _____.

(Fill in the blank)

Contd.

(b) Let $\underline{X} \sim N_P \left(\underline{\mu}, \underline{\Sigma} \right)$. Then the characteristic of $\underline{X}$ is given by

(i) $e^{i \underline{t}' \underline{\mu} + \frac{1}{2} \underline{t}' \underline{\Sigma} \underline{t}}$

(ii) $e^{i \underline{t}' \underline{\mu} - \frac{1}{2} \underline{t}' \underline{\Sigma} \underline{t}}$

(iii) $e^{i \underline{t}' \underline{\mu} + \frac{1}{2} \underline{t}' \underline{\Sigma} \underline{t}}$

(iv) None of the above

(Choose the correct option)

(c) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of:

(i) signs only

(ii) magnitudes only

(iii) sign and magnitude both

(iv) None of the above

(Choose the correct option)

(d) What is dispersion matrix in Multivariate data analysis?

(e) Let $(X, Y) \sim \text{BVND} (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.

Then state the conditional pdf of Y given $X = x$.

(f) What is run in non-parametric method ?

(g) Define Multiple correlation coefficient.

(h) Let $\underline{X} \sim N_3 \left(\underline{\mu}, \underline{\Sigma} \right)$. Given that

$$\underline{\Sigma} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 4 \end{pmatrix}$$

Are X_2 and X_3 independent ?

(i) The marginal distribution of a Bivariate normal distribution follows univariate normal distribution. (*State True or False*)

(j) The Kruskal-Wallis test is meant for :

(i) one way classification

(ii) two way classification

(iii) non classified data

(iv) None of the above

(*Choose the correct option*)

2. Answer **any four** of the following questions briefly: 2×4=8

(a) Define mean vector and dispersion matrix for multivariate data analysis.

- (b) State the marginal pdfs of X and Y in case of $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.
- (c) What assumptions are generally made for a non-parametric test?
- (d) Let $\underline{X} = (X_1 \ X_2 \ X_3)'$ have variance covariance matrix

$$\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$$

Find ρ_{12} .

- (e) Define marginal distribution of $X_1, X_2, \dots, X_k$ ($k < p$) in a p -variate multivariate analysis. Also define the conditional distribution of $X_{k+1}, X_{k+2}, \dots, X_p$ given $X_1, X_2, \dots, X_k$.
- (f) What indication can one get from the number of runs?
- (g) Give a brief idea of Principal component analysis.

(h) The pdf of bivariate normal distribution is

$$f(x, y) = k \exp \left[-\frac{1}{2(1-\rho^2)} (x^2 - 2\rho xy + y^2) \right],$$
$$-\infty < (x, y) < \infty$$

Find the constant k .

3. Answer **any three** of the following questions : 5×3=15

(a) If $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then show that X and Y are independent if and only if $\rho = 0$.

(b) Describe Kolmogorov-Smirnov one sample test stating its assumptions and hypotheses.

(c) Let $(X, Y) \sim \text{BVND}(0, 0, 1, 1, \rho)$. Then show that

$$Q = \frac{X^2 - 2\rho XY + Y^2}{(1 - \rho^2)}$$

is distributed as chi-square with 2d.f.

(d) Let $\hat{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$. Then find the distribution of $C\hat{X}$ where C is a $p \times p$ non-singular matrix of constant elements.

(e) Write an explanatory note on test of randomness.

(f) With usual notations, prove that

$$r_{12 \cdot 3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

(g) Examine if Hotelling's T^2 is invariant under changes in the units of measurement.

(h) Describe one sample sign test for testing the null hypothesis that the population median is a given value.

4. Answer **any three** questions from the following : 10×3=30

(a) (i) State *any two* applications of multivariate analysis. 2

(ii) Let $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Find the conditional distributions of $X/Y=y$ and $Y/X=x$. 8

(b) Derive the probability density function of p -variate normal distribution.

(c) (i) Describe the Wilcoxon Mann-Whitney U test. 5

(ii) Let $(X, Y) \sim \text{BVND}$ with parameters $\mu_x = 60$, $\mu_y = 75$, $\sigma_x = 5$, $\sigma_y = 12$ and $\rho = 0.55$. Then find $P\{65 \leq X \leq 75\}$ 5

(d) Let $\underline{X}_\alpha$ ($\alpha = 1, 2, \dots, N$) be a random sample from $N_p(\underline{\mu}, \underline{\Sigma})$ and let $\bar{\underline{X}} = \frac{1}{N} \sum_{\alpha=1}^N \underline{X}_\alpha$ be the sample mean vector.

Then prove that $\bar{\underline{X}}$ is distributed as $N_p\left(\underline{\mu}, \frac{\underline{\Sigma}}{N}\right)$.

(e) (i) Let $\underline{X}_\alpha^{(1)}$ ($\alpha = 1, 2, \dots, N_1$) be a random sample from $N_p(\underline{\mu}^{(1)}, \underline{\Sigma})$ and let $\underline{X}_\alpha^{(2)}$ ($\alpha = 1, 2, \dots, N_2$) be another random sample from $N_p(\underline{\mu}^{(2)}, \underline{\Sigma})$ where the common dispersion matrix $\underline{\Sigma}$ is unknown. Discuss the procedure to test the hypothesis $H_0 : \underline{\mu}^{(1)} = \underline{\mu}^{(2)}$ using Hotelling's T^2 statistic. 5

(ii) In what way the ordinary sign test can be performed for paired samples? Explain. 5

(f) (i) State *any two* properties of multivariate normal distribution.

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(ii) Derive the bivariate normal density as a particular case of multivariate normal distribution.

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(g) (i) Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$. Find the distribution of

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$$\begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$$

(ii) Derive the formula for Multiple correlation coefficient for a trivariate distribution.

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(h) (i) Explain the distribution free method.

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(ii) Derive the moment generating function of a bivariate normal distribution with usual parameters.

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